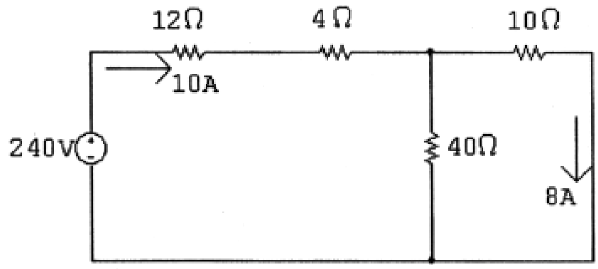
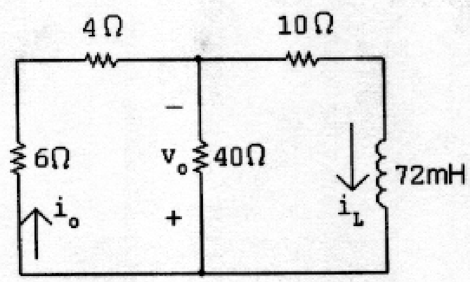
Homework 7
Solution

P 7.14 $t < 0$:



$i_L(0^+) = 8\,\text{A}$

$t > 0$:



$$R_e = \frac{(10)(40)}{50} + 10 = 18\,\Omega$$

$$\tau = \frac{L}{R_e} = \frac{0.072}{18} = 4\,\text{ms}; \qquad \frac{1}{\tau} = 250$$

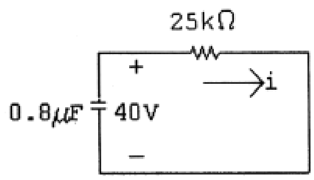
$$\therefore \quad i_L = 8e^{-250t}\,\text{A}$$

$$\therefore \quad v_o = -10i_L - 0.072\frac{di_L}{dt} = -80e^{-250t} + 144e^{-250t}$$

$$= 64e^{-250t}\,\text{A} \quad t \geq 0^+$$

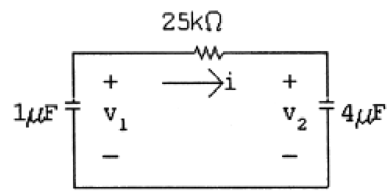
P 7.21 **[a]** $v_1(0^-) = v_1(0^+) = 40\,\text{V}$ $\qquad v_2(0^+) = 0$

$C_{eq} = (1)(4)/5 = 0.8\,\mu\text{F}$



$\tau = (25 \times 10^3)(0.8 \times 10^{-6}) = 20\text{ms}; \qquad \frac{1}{\tau} = 50$

$i = \frac{40}{25{,}000}e^{-50t} = 1.6e^{-50t}\,\text{mA}, \qquad t \geq 0^+$



$$v_1 = \frac{-1}{10^{-6}}\int_0^t 1.6 \times 10^{-3}e^{-50x}\,dx + 40 = 32e^{-50t} + 8\,\text{V}, \qquad t \geq 0$$

$$v_2 = \frac{1}{4 \times 10^{-6}}\int_0^t 1.6 \times 10^{-3}e^{-50x}\,dx + 0 = -8e^{-50t} + 8\,\text{V}, \qquad t \geq 0$$

[b] $w(0) = \frac{1}{2}(10^{-6})(40)^2 = 800\,\mu\text{J}$

[c] $w_{\text{trapped}} = \frac{1}{2}(10^{-6})(8)^2 + \frac{1}{2}(4 \times 10^{-6})(8)^2 = 160\,\mu\text{J}.$

The energy dissipated by the 25 kΩ resistor is equal to the energy dissipated by the two capacitors; it is easier to calculate the energy dissipated by the capacitors (final voltage on the equivalent capacitor is zero):

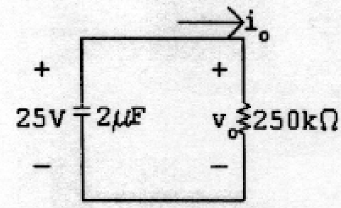
$w_{\text{diss}} = \frac{1}{2}(0.8 \times 10^{-6})(40)^2 = 640\,\mu\text{J}.$

Check: $w_{\text{trapped}} + w_{\text{diss}} = 160 + 640 = 800\,\mu\text{J}; \qquad w(0) = 800\,\mu\text{J}.$

P 7.30 **[a]** $C_e = \dfrac{(2+1)6}{2+1+6} = 2\,\mu\text{F}$

$$v_o(0) = -5 + 30 = 25\,\text{V}$$

$$\tau = (2 \times 10^{-6})(250 \times 10^{3}) = 0.5\,\text{s}; \qquad \frac{1}{\tau} = 2$$



$$v_o = 25e^{-2t}\,\text{V}, \qquad t > 0^+$$

[b] $w_o = \dfrac{1}{2}(3 \times 10^{-6})(30)^2 + \dfrac{1}{2}(6 \times 10^{-6})(5)^2 = 1425\,\mu\text{J}$

$$w_{\text{diss}} = \frac{1}{2}(2 \times 10^{-6})(25)^2 = 625\,\mu\text{J}$$

$$\%\ \text{diss} = \frac{1425 - 625}{1425} \times 100 = 56.14\%$$

[c] $i_o = \dfrac{v_o}{250 \times 10^{-3}} = 100e^{-2t}\,\mu\text{A}$

$$v_1 = -\frac{1}{6 \times 10^{-6}}\int_0^t 100 \times 10^{-6} e^{-2x}\,dx - 5 = -16.67\int_0^t e^{-2x}\,dx - 5$$

$$= -16.67\frac{e^{-2x}}{-2}\Big|_0^t - 5 = 8.33e^{-2t} - 13.33\,\text{V} \qquad t \geq 0$$

[d] $v_1 + v_2 = v_o$

$$v_2 = v_o - v_1 = 25e^{-2t} - 8.33e^{-2t} + 13.33 = 16.67e^{-2t} + 13.33\,\text{V} \quad t \geq 0$$

[e] $w_{\text{trapped}} = \dfrac{1}{2}(6 \times 10^{-6})(13.33)^2 + \dfrac{1}{2}(3 \times 10^{-6})(13.33)^2 = 800\,\mu\text{J}$

$$w_{\text{diss}} + w_{\text{trapped}} = 625 + 800 = 1425\,\mu\text{J} \qquad \text{(check)}$$

P 7.36 **[a]** $v_o(0^+) = -I_g R_2; \qquad \tau = \frac{L}{R_1 + R_2}$

$$v_o(\infty) = 0$$

$$v_o(t) = -I_g R_2 e^{-[(R_1+R_2)/L]t}\ \text{V}, \qquad t \geq 0^+$$

[b] $v_o = -(10)(15)e^{-\frac{(5+15)}{0.016}t} = -150e^{-1250t}\ \text{V}, \qquad t \geq 0^+$

[c] $v_o(0^+) \to \infty$, and the duration of $v_o(t) \to$ zero

[d] $v_{sw} = R_2 i_o; \qquad \tau = \frac{L}{R_1 + R_2}$

$$i_o(0^+) = I_g; \qquad i_o(\infty) = I_g \frac{R_1}{R_1 + R_2}$$

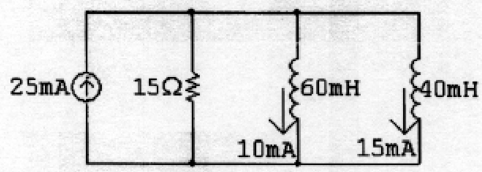
Therefore $\quad i_o(t) = \frac{I_g R_1}{R_1+R_2} + \left[I_g - \frac{I_g R_1}{R_1+R_2}\right] e^{-[(R_1+R_2)/L]t}$

$$i_o(t) = \frac{R_1 I_g}{(R_1+R_2)} + \frac{R_2 I_g}{(R_1+R_2)} e^{-[(R_1+R_2)/L]t}$$

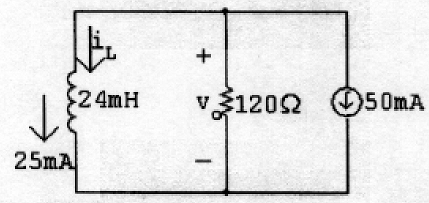
Therefore $\quad v_{sw} = \frac{R_1 I_g}{(1+R_1/R_2)} + \frac{R_2 I_g}{(1+R_1/R_2)} e^{-[(R_1+R_2)/L]t}, \qquad t \geq 0^+$

[e] $|v_{sw}(0^+)| \to \infty; \qquad$ duration $\to 0$

P 7.44 **[a]** $t < 0$



$t > 0$



$$i_L(0^-) = i_L(0^+) = 25\,\text{mA}; \qquad \tau = \frac{24 \times 10^{-3}}{120} = 0.2\,\text{ms}; \qquad \frac{1}{\tau} = 5000$$

$$i_L(\infty) = -50\,\text{mA}$$

$$i_L = -50 + (25 + 50)e^{-5000t} = -50 + 75e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

$$v_o = -120[75 \times 10^{-3}e^{-5000t}] = -9e^{-5000t}\,\text{V}, \qquad t \geq 0^+$$

[b] $$i_1 = \frac{1}{60 \times 10^{-3}}\int_0^t -9e^{-5000x}\,dx + 10 \times 10^{-3} = (30e^{-5000t} - 20)\,\text{mA}, \qquad t \geq 0$$

[c] $$i_2 = \frac{1}{40 \times 10^{-3}}\int_0^t -9e^{-5000x}\,dx + 15 \times 10^{-3} = (45e^{-5000t} - 30)\,\text{mA}, \qquad t \geq 0$$